Lepton flavor violating signals of the neutral top-pion in future lepton colliders

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Abstract

The presence of the top-pions $\pi_t^{0,\pm}$ in the low-energy spectrum is an inevitable feature of the topcolor scenario. Taking into account the constraints of the present experimental limit of the lepton flavor violating(LFV) process $\mu \to e\gamma$ on the free parameters of topcolor-assisted techicolor(TC2) models, we study the contributions of the neutral top-pion π_t^0 to the LFV processes $\mu^+\mu^- \to \tau\mu$ (or τe), $\gamma\gamma \to \tau\mu$ (or τe), $e^+e^- \to \tau\mu$, and $e\gamma \to e\pi_t^0 \to e\tau\mu(e)$ via the flavor changing (FC) couplings $\pi_t^0 l_i l_j$ and discuss the possibility of searching for the LFV signals via these processes in future lepton colliders.

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I. Introduction

The solar neutrino experiments[1] and the atmospheric neutrino experiments[2] confirmed by reactor and accelerator experiments[3] have made one believe that neutrinos are massive and oscillate in flavors, and provide the only direct observation of physics that cannot be accommodated within the standard model(SM), which can be seen as the first experimental clue for the existence of new physics beyond the SM. Thus, the SM requires some modification to account for the pattern of neutrino mixing, in which the lepton flavor violating(LFV) processes are allowed. The observation of the LFV signals in present or future high-energy experiments would be a clear signature of new physics beyond the SM.

Many kinds of popular specific models beyond the SM predict the presence of new particles, such as new gauge bosons and new scalars, which can naturally lead to the tree-level LFV couplings. In general, these new particles could enhance branching ratios for some LFV processes, and perhaps bringing them into the observable threshold of the present and next generations of collider experiments. Furthermore, nonobservability of these LFV processes can lead to strong constraints on the nature of new physics. Thus, studying the possible LFV signals of new particles in various high-energy colliders is very interesting and needed.

To completely avoid the problems arising from the elementary Higgs field in the SM, various kinds of dynamical electroweak symmetry breaking ($EWSB$) models have been proposed, and among which the topcolor scenario is attractive because it can explain the large top quark mass and provide possible $EWSB$ mechanism[4]. Almost all of these kind of models propose that the scale of the gauge groups should be flavor nonuniversal. When one writes the nonuniversal interactions in the masseigen basis, it can induce the tree -level flavor changing(FC) couplings, which can generate rich phenomenology.

The presence of the physical top-pions ${\pi_t}^{0,\pm}$ in the low-energy spectrum is an inevitable feature of the topcolor scenario, regardless of the dynamics responsible for $EWSB$ and other quark masses[5]. One of the most interesting features of the top-pions ${\pi_t}^{0,\pm}$ is that they have large Yukawa couplings to the third-generation quarks and can induce

the tree-level FC couplings in quark sector and in the lepton sector. The obviously phenomenological implication of this feature is that the top-pions can be significant produced via some FC processes, and their possible signatures might be observed in future hadron colliders[5,6] and lepton colliders[7,8]. Furthermore, the top-pions can generate large corrections to some observables related with the FC couplings and give characteristic signatures at various high-energy colliders[9,10]. On the other hand, the top-pions can generate significant contributions to the LFV processes, such as $l_i \to l_j\gamma$, $l_i \to l_j l_k l_l$ and $Z \to l_i l_j$ [11]. The branching ratios for some of these processes can be enhanced to their current or future experimental bounds, which can give severe constraints on the free parameters of topcolor scenario.

Among the various LFV processes that have been considered in the literature, the most fruitful ones are the radiative decays $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\eta$, and $\mu \to e\gamma$, since their branching ratios are tested with high precision[12,13]. These processes usually provide the restrictive experimental bounds on the free parameters of the popular specific models beyond the SM. For example, the contributions of the Higgs bosons to these processes have been extensively studied in Refs.[14,15]. Taking into account the constraints of the experimental upper limits of these LFV processes on the free parameters, the LFV decays of the Higgs bosons and possible signals in the future high-energy collider experiments have been studied in Ref.[16].

In this paper, we will study the LFV signals of the neutral top-point π_t^0 at the future various lepton colliders in the context of the topcolor-assisted technicolor ($TC2$) models[17] with free parameters being compatible with the most relevant data of μ radiative decay $\mu \to e\gamma$. In this work, we consider the contributions of the neutral top-pion π_t^0 to the LFV processes $\mu^+\mu^- \to \tau\mu$ (or τe), $\gamma\gamma \to \tau\mu$ (or τe), $e^+e^- \to \tau\mu$, and $e\gamma \to e\pi_t^0 \to e\tau\mu(e)$ via the FC couplings $\pi_t^0 l_i l_j$ and discuss possibility of directly searching for the LFV signals in the future lepton colliders via these processes. For completeness, we also include an estimation of the contributions of the new gauge boson Z' predicted by $TC2$ models to these processes and compare them with those of the neutral top-pion π_t^0.

This paper is organized as follows. Section II contains a short summary of the relevant coupling to ordinary particles and decay modes of neutral top-pion π_t^0 in $TC2$ models. Discussions of the constraints coming from the LFV τ and μ decays, the muon anomalous magnetic moment a_μ, and other observables on the relevant free parameters are also given in this section. Section III, IV and V are devoted to the computation of the produce cross sections generated by π_t^0 exchange for the LFV processes $\mu^+\mu^- \to \tau\mu(\tau e), \gamma\gamma \to \tau\mu(\tau e), e^+e^- \to \tau\mu(\tau e)$, and $e\gamma \to e\tau\mu(e\tau e)$, respectively. Some phenomenological analyses are also included. Our conclusions are given in Sec. VI.

II. The LFV coupling of π_t^0 to ordinary particles

For $TC2$ models [17], technicolor(TC) interactions play a main role in breaking the electroweak symmetry. Topcolor interactions make small contributions to $EWSB$ and give rise to the main part of the top quark mass, $(1-\varepsilon)m_t$, with the parameter $\varepsilon << 1$, Thus, there is the following relation:

$$\nu_\pi^2 + F_t^2 = \nu_w^2, \tag{1}$$

where ν_π represents the contributions of TC interactions to $EWSB$, $\nu_w = \nu/\sqrt{2} \simeq$174GeV. Here $F_t \simeq$50GeV is the physical top-pion decay constant. This means that the masses of the SM gauge bosons W and Z are given by absorbing the linear combination of the top-pions and technipions. The orthogonal combination of the top-pion and technipions remains unabsorbed and physical[18,19,20]. However, the absorbed Goldstone boson linear combination is mostly the technipions, while the physical combination is mostly the top-pions, which are usually called physical top-pions($\pi_t^{0,\ \pm}$).

For $TC2$ models, the underlying interactions, topcolor interactions, are nonuniversal and therefore do not possess Glashow-Iliopoulos-Maiani(GIM) mechanism. The nonuniversal gauge interactions result in the new FC coupling vertices when one writes the interactions in the mass eigenbasis. Thus, the top-pions can induce the new FC coupling vertices. The couplings of the neutral top-pion π_t^0 to ordinary fermions, which are related to our calculation, can be written as [5,8,11,17]:

$$\frac{m_t}{\sqrt{2}F_t}\frac{\sqrt{\nu_w^2-F_t^2}}{\nu_w}[K_{UR}^{tt}K_{UL}^{tt^*}\bar{t}\gamma^5t\pi_t^0+\frac{m_b-m_b'}{m_t}\bar{b}\gamma^5b\pi_t^0+K_{UR}^{tc}K_{UL}^{tt^*}\bar{t_L}c_R\pi_t^0]$$
$$+\frac{m_l}{\sqrt{2}\nu_w}\bar{l}\gamma^5l\pi_t^0+\frac{m_\tau}{\sqrt{2}\nu_w}K_{\tau i}\bar{\tau}\gamma^5l_i\pi_t^0, \tag{2}$$

where $m_b' \approx 0.1\varepsilon m_t$ is the part of the bottom-quark mass generated by extended technicolor(ETC)[9]. K_{UL} and K_{UR} are rotation matrices that diagonalize the up-quark mass matrix M_U, i.e. $K_{UL}^{+}M_UK_{UR}=M_U^{dia}$. To yield a realistic form of the Cabibbo-Kobayashi-Maskawa(CKM) matrix V, it has been shown that their values can be taken as [5]:

$$K_{UL}^{tt}\approx 1, \quad K_{UR}^{tt}=1-\varepsilon, \quad K_{UR}^{tc}\leq\sqrt{2\varepsilon-\varepsilon^2}\ . \tag{3}$$

In the following calculation, we will take $K_{UR}^{tc}=\sqrt{2\varepsilon-\varepsilon^2}$ and take ε as a free parameter, which is assumed to be in the range of 0.01-0.1[4,17]. $l=\tau,\mu$ or e, l_i(i=1,2) is the first(second)generation lepton $e(\mu)$, and $k_{\tau i}$ is the flavor mixing factor between the third- and the first-or second- generation leptons. Certainly, there is also the FC scalar coupling $\pi_t^0\mu e$. However, the topcolor interactions only contact with the third-generation fermions, and thus, the flavor mixing between the first- and second-generation fermions is very small, which can be ignored.

The limits on the top-pion mass m_{π_t} might be obtained via studying its effects on various experimental observables. For example, considering the couplings of the neutral top-pion π_t^0 to bottom quark through instanton effects, Ref.[18] has shown that the process $b\rightarrow s\gamma$, $B-\bar{B}$ mixing and $D-\bar{D}$ mixing demand that the top-pions are likely to be light, with masses of the order of a few hundred GeV. Since the negative top-pion corrections to the $Z\rightarrow b\bar{b}$ branching ratio R_b become smaller when the top-pion is heavier, the electroweak precision measurement data of R_b give rise to a certain lower bound on the top-pion mass. It was shown that the top-pion mass should not be lighter than the order of $1TeV$ to make TC2 models consistent with the electroweak precision measurement data[19]. Reference [20] restudied this problem and found that the top-pion mass m_{π_t} is allowed to be in the range of a few hundred GeV depending on the models. Thus, the

value of the top-pion mass m_{π_t} remains subject to large uncertainty[4]. In general, the top-pion mass m_{π_t} is allowed to be in the range of a few hundred GeV depending on the models. In this paper, we will assume $150GeV \leq m_{\pi_t} \leq 350GeV$. In this case, the possible decay modes of π_t^0 are $b\overline{b}$, $\overline{t}c$, $\overline{f}f$(f is the first-or second-generation fermions, or the third-generation bottom quark or leptons), gg, $\gamma\gamma$ and τl, ($l = \mu$ or e). The total decay width which has been calculated in Ref.[21]. However, the modes of decay into leptons are not included in this reference. If we take into account these decay modes, then the branching ratio $Br(\pi_t^0 \to \tau\mu)$ is in the range of $7.2 \times 10^{-3}k_{\tau\mu}^2 \sim 2.5 \times 10^{-5}k_{\tau\mu}^2$ for $150GeV \leq m_{\pi_t} \leq 350GeV$ and $0.01 \leq \varepsilon \leq 0.1$.

Since the topcolor interactions only contact with the third- generation fermions and the flavor mixing between the first- and second-generation fermions is very small, the B system observables cannot give significant constraints on the flavor mixing factor $k_{\tau i}$.

It is well known that the precision measurement of the muon anomalous magnetic moment a_μ is a sensitive test for the new physics beyond the SM. Comparing the new measurement value of a_μ with the present SM prediction, there remains a tantalizing discrepancy[22]:

$$a_\mu^{exp} - a_\mu^{SM} = (24.5 \pm 9) \times 10^{-10}. \tag{4}$$

If we assume that the observed deviation, as shown in $Eq.(4)$, comes from the contributions of new particles predicted by $TC2$ models, then we might obtain a constraint on $TC2$ models using this deviation[23]. However, we find that the contributions of $TC2$ models to a_μ come mainly from the ETC gauge bosons and nonuniversal gauge boson Z'; the contributions of π_t^0 are smaller than 1×10^{-13} i.e. $\delta a_\mu^{\pi_t^0} < 1 \times 10^{-13}$. Thus, the recently measurement value of a_μ can not give significant constraints on the free parameters, which are related to the top-pion, such as ε, m_{π_t}, and k_{τ_i}.

The neutral top-pion π_t^0 can produce significant contributions to the LFV processes $l_i \to l_j\gamma$, $l_i \to l_jl_kl_l$, and $Z \to l_il_j$ via the FC couplings $\pi_t^0\tau\mu$ and $\pi_t^0\tau e$, and can enhance the branching ratios of these processes by several orders of magnitude[11]. For the processes $l_i \to l_j\gamma$, the contributions come from the on-shell photon penguin diagrams, while the contributions come from both the on-shell photon penguin diagrams and the

tree-level diagrams for the processes $\tau \to l_i l_j l_k$. Reference [11] has shown that, in all of the parameter space of $TC2$ models, the branching ratios of the processes $\tau \to l\gamma$, $l_i \to l_j l_k l_l$, and $Z \to l_i l_j$ are far below the experimental upper bound on these processes, except for the process $\mu \to 3e$, which might approach the observable threshold of near-future experiments[13]. The present experimental limit of the process $\mu \to e\gamma$ can give severe constraints on the free parameters of $TC2$ models. If we assume $k = k_{\tau\mu} = k_{\tau e}$ and $150GeV \leq 350GeV$, then there must be $k \leq 0.16$. Taking into account this constraint, we will study possible LFV signals of the neutral top-pion π_t^0 at future various lepton colliders in the following sections.

III. LFV signals of the neutral top-pion π_t^0 in the future muon collders

A muon collider is an excellent tool to study the properties of a heavy scalar or pseudoscalar and potential new physics effects[24]. It has been shown that a large number of Higgs bosons[24] or new particles, such as technihadron and technipions[25], can be produced through s-channel resonance processes. The FC scalar couplings[26,27] might be tested at future muon colliders. Thus, the future muon collider opens up the possibility of studying the LFV signals mediated by the new scalars.

The neutral top-pion π_t^0 has FC couplings to the leptons at tree level and thus can contribute to the processes $\mu^+\mu^- \to \tau\mu(\tau e)$ via π_t^0 exchange in the s-channel. In spite of the fact that the coupling $\pi_t^0\mu^+\mu^-$, being proportional to m_μ/v, is very small, if the muon collider with the center-of-mass($c.m.$) energy $\sqrt{s}$ runs on π_t^0 resonance ($\sqrt{s} = m_{\pi_t}$), then the neutral top-pion π_t^0 may be produced at an appreciable rate[25, 27]. Thus, the neutral top-pion might give observable LFV signals at future muon collider experiments.

After averaging over initial polarization and integrating over the scattering angle, it is straightforward to obtain the unpolarized cross section of the process $\mu^+\mu^- \to \tau\mu(\tau e)$ mediated by π_t^0 exchange. The s-channel resonance cross section for this process can be approximately written as:

$$\sigma(\tau\mu(e)) \approx \frac{4\pi}{m_{\pi_t}} \cdot Br(\pi_t^0 \to \mu^+\mu^-) Br(\pi_t^0 \to \tau\mu(e)), \qquad (5)$$

where $Br(\pi_t^0 \to \mu^+\mu^-)$ and $Br(\pi_t^0 \to \tau\mu(e))$ represent the branching ratios of π_t^0 decaying

to $\mu^+\mu^-$ and $\tau\mu(e)$, respectively. Observably, there is $\sigma(\tau\mu) \approx \sigma(\tau e)$ for $k_{\tau\mu} = k_{\tau e}$ and neglecting the final state masses, which is different from that generated by the Higgs bosons[26]. In that case, compared the production cross section $\sigma(\tau\mu)$, the cross section $\sigma(\tau e)$ is suppressed by a factor of $m_e/m_\mu \sim 1/200$.

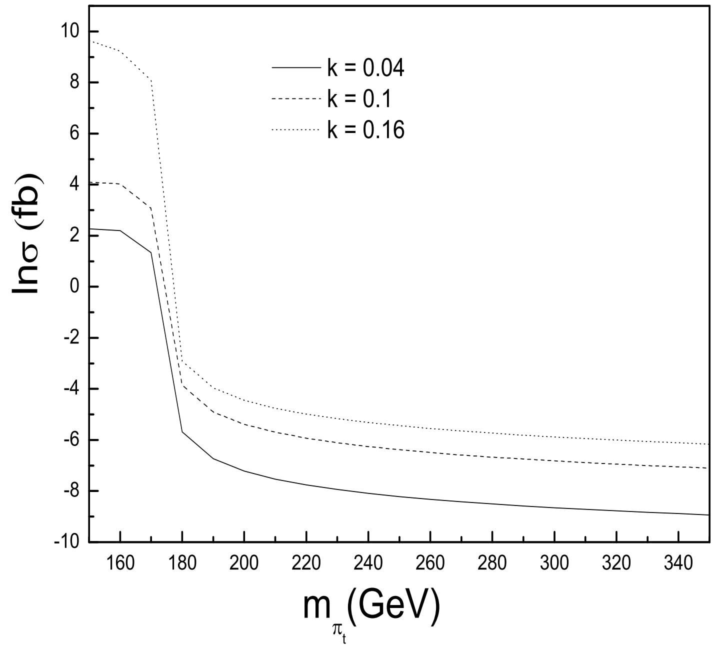

Figure 1: The resonance production cross section $\sigma(\tau\mu)$ as function of the top-pion mass m_{π_t} for $\sqrt{s} = m_{\pi_t}$ and three values of the mixing parameter k.

In $Fig.1$ we show the resonance production cross section $\sigma(\tau\mu)$ as a function of the top-pion mass m_{π_t} for $\sqrt{s} = m_{\pi_t}$ and three values of the mixing parameter k. Since the cross section $\sigma(\tau\mu)$ is not sensitive to the free parameter ε, we have taken $\varepsilon = 0.05$ in $Fig.1$. One can see from $Fig.1$ that the $\sigma(\tau\mu)$ decreases with m_{π_t} increasing and the mixing parameter k decreasing. For $m_{\pi_t} \geq m_t + m_c$, the FC channel $\pi_t^0 \rightarrow \bar{t}c$ opens up and the branching ratios $Br(\pi_t^0 \rightarrow \mu^+\mu^-)$ and $Br(\pi_t^0 \rightarrow \tau\mu)$ drop substantially, and thus the cross sections $\sigma(\tau\mu)$ and $\sigma(\tau e)$ drop considerably. The values of $\sigma(\tau\mu)$ are in the ranges of $1.54 \times 10^2 \sim 3.4 \times 10^{-2} fb$ and $5.5 \times 10^{-2} \sim 1.5 \times 10^{-4} fb$ for $k \leq 0.16$, $150GeV \leq m_{\pi_t} < 180GeV$, and $180GeV \leq m_{\pi_t} \leq 350GeV$, respectively. If we assume

a future muon collider runing with the $c.m.$ energy $\sqrt{s} = 200 \sim 500GeV$ and a yearly integrated luminosity of $\mathcal{L} = 20fb^{-1}$, then there will be several and up to thousand $\tau\mu$(or τe) events to be generated a year for $m_{\pi_t} < 180GeV$.

For the LFV processes $\mu^+\mu^- \to \tau\mu($ or $\tau e)$, the final leptons always emerge back to back and carrying a constant energy which is one half of the $c.m.$ energy $\sqrt{s}$. The main background would arise from the process $\mu^+\mu^- \to \overline{\tau}\mu\overline{\nu_\mu}\nu_\tau$ or $\overline{\tau}e\overline{\nu_e}\nu_\tau$. It has been shown that the contributions to the background come mainly from the diagrams with Higgs boson exchange[26]. The background cross section can only reach a peak around $m_H = 130GeV$ and then drops quickly out of this range. Thus, as long as $150GeV \leq m_{\pi_t} \leq m_t + m_c$, the LFV signals of the neutral top pion π_t^0 should be observed via the resonance processes $\mu^+\mu^- \to \tau\mu$(or τe) in the future muon colliders.

$TC2$ models also predict the existence of a nonuniversal $U(1)$ gauge boson Z', which can lead to the tree-level FC couplings $Z'\tau\mu$ and $Z'\tau e$. Thus, the nonuniversal gauge boson Z' has contributions to the LFV processes $\mu^+\mu^- \to \tau\mu($ or $\tau e)$ via the s-channel Z' exchange. At leading order, the unpolarized cross section $\sigma'(\tau\mu)$ generated by Z' exchange can be written as:

$$\sigma^{'}(\tau\mu) = \frac{25\pi^2\alpha^2\lambda_{\tau\mu}^2}{12C_W^6K_1}\frac{s}{(s - M_{Z'}^2)^2 + M_{Z'}^2\Gamma_{Z'}^2}, \tag{6}$$

where $C_W = \cos\theta_W$, θ_W is the Weinberg angle, $\lambda_{\tau\mu}$ is the coupling constant of the FC vertex $Z'\tau\mu$. K_1 is the mixing parameter between the nonuniversal gauge boson Z' and the SM gauge boson Z. Using $Eq.(6)$, we can easily give the numerical results. However, considering the constraints of the electroweak precision measurement data on the Z' mass $M_{Z'}(M_{Z'} > 1TeV)$[4,28], the cross section $\sigma'(\tau\mu)$ is smaller than $10^{-4}fb$ in all of the parameter space at the future muon colliders with $\sqrt{s} = 300 \sim 500GeV$, which cannot produce observable signals. Furthermore, even if the nonuniversal gauge boson Z' can produce observable LFV signals, we can use polarized beams to separate the contributions from π_t^0 exchange from those from Z' exchange.

IV. The neutral top-pion π_t^0 and the LFV processes $\gamma\gamma \to \pi_t^0 \to \tau\mu(\tau e)$

It is widely believed that hadron colliders, such as Tevatron and future LHC, can directly probe possible new physics beyond the SM up to a few TeV, while the future

high-energy linear e^+e^- collider (ILC) is also required to complement the probe of the new particles with detailed measurement. A future ILC will offer an excellent opportunity to study production and decay of the new particles with percent-level-precision[29]. A unique feature of the future ILC is that it can be transformed to $\gamma\gamma$ colliders with the photon beams generated by the Compton backward scattering of the initial electron and laser beams. Their effective luminosity and energy are expected to be comparable to those of the ILC. In some scenarios, they are the best instrument for the discovery of signals of new physics[30].

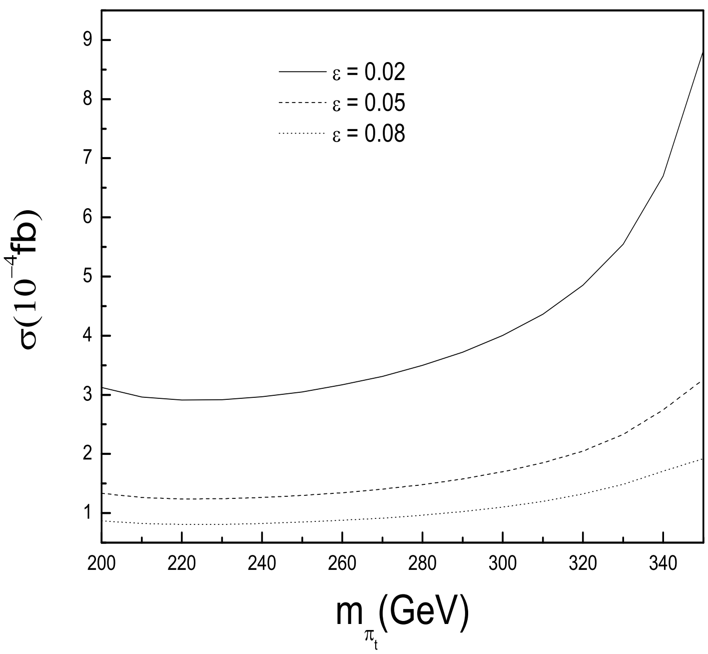

Figure 2: The effective cross section $\sigma(s)$ as a function of m_{π_t} for $\sqrt{s} = 500GeV, k = 0.1$, and three values of the free parameter ε.

Similar to the contributions of the neutral top-pion π_t^0 to top-charm associated production at $\gamma\gamma$ colliders[7,9], the contributions of π_t^0 to the LFV processes $\gamma\gamma \rightarrow \tau\mu(\tau e)$ via the FC couplings $\pi_t^0 l_i l_j$ proceed through the self-energy diagrams, vertex diagrams and the s-channel triangle diagram. We have explicitly calculated the contributions of these Feynman diagrams and found that the dominant contributions come from the s-

channel triangle diagram, whereas the remaining diagrams give negligible contributions. Our numerical results are shown in $Fig.2$, in which we plot the effective cross section $\sigma(s) = \sigma(e^+e^- \to \gamma\gamma \to \tau\mu)$ for the LFV process $\gamma\gamma \to \tau\mu$ as a function of the top-pion mass m_{π_t} for the $c.m.$ energy $\sqrt{s} = 500GeV$, the flavor mixing factor $k = 0.1$, and three values of the parameter ε. From this figure, one can see that the cross section $\sigma(s)$ for the process $e^+e^- \to \gamma\gamma \to \tau\mu$ (or τe) is relatively insensitive to the parameter ε. As the top-pion mass m_{π_t} increases, the cross section increases monotonically. For $k = 0.1$ and $200GeV \leq m_{\pi_t} \leq 350GeV$, the value of the cross section $\sigma(e^+e^- \to \gamma\gamma \to \tau\mu)$ is smaller than $1 \times 10^{-3} fb$. Certainly, if we assume $150GeV \leq m_{\pi_t} < 180GeV$, then the cross section increases rapidly and its value can reach $8.3 \times 10^{-2} fb$.

The flavor mixing parameter k controls the strength of the FC coupling $\pi_t^0 \tau l (l = \mu$ or $e)$ and further determines the LFV signals of the neutral top-pion π_t^0. Its value is severely constrained by the current experimental upper bound on the LFV process $\mu \to e\gamma$[11]. There is $k \leq 0.16$ for $m_{\pi_t} \leq 350GeV$. Taking into account this constraint, we plot the effective cross section $\sigma(s)$ as a function of k for $\sqrt{s} = 500GeV$, $\varepsilon = 0.05$, and three values of the top-pion mass m_{π_t} in $Fig.3$.

From the above discussion, we can see that, if we assume $200GeV \leq m_{\pi_t} < 350GeV$, then the effective cross section $\sigma(s)$ is smaller than $1 \times 10^{-3} fb$ in most of the parameter space consistent with the constraint from the LFV process $\mu \to e\gamma$. However, for $150GeV \leq m_{\pi_t} < 180GeV$, $\sigma(s)$ is in the range of $1 \times 10^{-2} fb \sim 8 \times 10^{-2} fb$. In this case, there will be several tens $\tau\mu(e)$ events to be generated a year at the future ILC with $\sqrt{s} = 500GeV$ and the yearly integrated $\mathcal{L}_{int} = 340 fb^{-1}$, which might be observed in future ILC experiments.

The neutral top-pion π_t^0 can also contribute to the LFV processes $e^+e^- \to \tau\mu(\tau e)$ via the FC couplings $\pi_t^0 \tau\mu(e)$. The relevant Feynman diagrams are shown in $Fig.4$. Since the s-channel process $e^+e^- \to \gamma^*,\ Z^* \to \tau\mu$ is suppressed by the propagators of the intermediate photon or Z-boson in the future ILC with high center-of-mass energy, the cross section of the process $e^+e^- \to \tau\mu$ generated by the neutral top-pion π_t^0 should be smaller than that of the process $e^+e^- \to \gamma\gamma \to \tau\mu$. We have confirmed this expectation

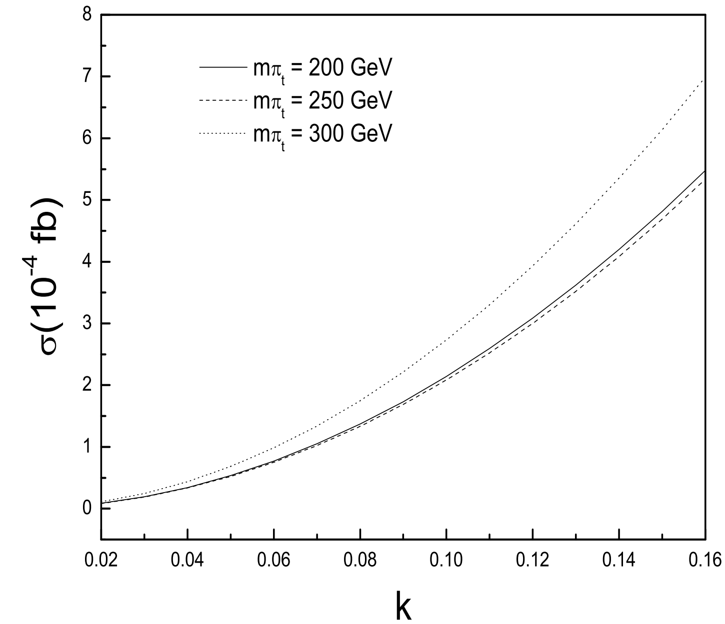

Figure 3: The effective cross section $\sigma(s)$ as a function of k for $\sqrt{s} = 500GeV$, $\varepsilon = 0.05$ and three values of the π_t^0 mass m_{π_t}.

through explicit calculation. Our numerical results show that the cross section for the process $e^+e^- \to \tau\mu$ at the ILC with $\sqrt{s} = 500GeV$ is smaller than $1 \times 10^{-7} fb$ in all of the parameter space of $TC2$ models, which can not be detected in future ILC experiments.

The production cross section of the process $e^+e^- \to Z' \to \tau\mu(\tau e)$ is approximately equal to that of the process $\mu^+\mu^- \to Z' \to \tau\mu(\tau e)$, which is smaller than $1 \times 10^{-4} fb$ in all of the parameter space of the $TC2$ models.

V. The neutral top-pion π_t^0 and the LFV process $e^-\gamma \to e^-\tau\mu$

A future ILC can also operate in $e^-\gamma$ collisions, where the γ beam is generated by the Compton backward scattering of the incident position and laser beam and its energy and luminosity can reach the same order of magnitude of the corresponding position beam[30]. The $e^-\gamma$ collisions can produce particles which are kinematically not accessible at the e^+e^- collisions at the same colliders. Thus, the $e^-\gamma$ collisions are well suite for studying the production and decays of new particles. In this section, we will discuss the contributions of the neutral top-pion π_t^0 to the LFV process $e^-\gamma \to e^-\tau\mu$ and see whether the LFV

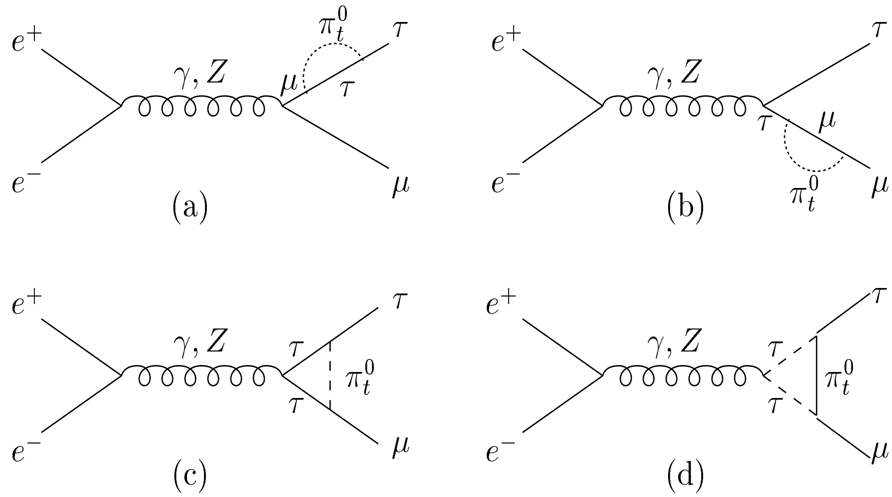

Figure 4: Feynman diagrams for the process $e^+e^- \to \tau\mu$ contributed by the LFV coupling $\pi_t^0\tau\mu$

signals of π_t^0 can be detected via $e^-\gamma$ collisions in the future ILC.

In the $TC2$ models, $e^-\tau\mu$ production in $e^-\gamma$ collisions proceeds through the process $e^-\gamma \to e^-\gamma^*\gamma \to e^-\pi_t^0 \to e^-\tau\mu$, in which the γ beam is generated by the backward Compton scatting of incident positron and laser beam and the γ^* beam is radiated from the e^- beam. The relevant Feynman diagram is shown in $Fig.5$. In our calculation, we use the $Weizsäclcer - Williams$ approximation[31] and treat the virtual photon γ^* coming from the e^- beam as a real photon. In this case, the effective cross section of the LFV process $e^-\gamma \to e\tau\mu$ in the ILC experiments can be obtained by folding the cross section of the subprocess $\gamma^*\gamma \to \tau\mu$ with the backscattered photon distribution function $F_{\gamma/e}(x)$[32] and the function $P_{\gamma/e}(x, E_e)$, which is the probability of finding a photon with a fraction x of energy E_e in an ultrarelativistic electron[31].

Our numerical results are shown in $Fig.6$, in which we plot the cross section $\sigma(s)$ for the LFV process $e^+e^- \to e^-\gamma \to e^-\gamma\gamma^* \to e^-\tau\mu$ as a function of m_{π_t} for $\sqrt{s} = 500GeV$ and three values of the mixing parameter k. Since the cross section $\sigma(s)$ is not sensitive to the parameter ε, we have assumed $\varepsilon = 0.05$ in $Fig.6$. One can see from $Fig.6$ that, for

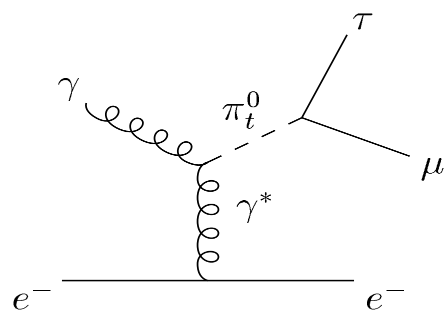

Figure 5: Feynman diagram for the process $e^-\gamma \to e^-\tau\mu$ contributed by the LFV coupling $\pi_t^0\tau\mu$

$180GeV \leq m_{\pi_t} \leq 350GeV$, the value of the cross section $\sigma(s)$ is smaller than $4 \times 10^{-5} fb$. Similar to the case for the process $e^+e^- \to \gamma\gamma \to \tau\mu$, the production cross section of this process for $150GeV \leq m_{\pi_t} < 180GeV$ is significantly larger than that for $180GeV \leq m_{\pi_t} \leq 350GeV$. For $150GeV \leq m_{\pi_t} < 180GeV$, the effective cross section of the process $e^-\gamma \to e^-\tau\mu$ can reach $4.1 \times 10^{-3} fb$. Even in this case, i.e. $150GeV \leq m_{\pi_t} < 180GeV$, the neuatral top-pion π_t^0 cannot produce observable signals via the LFV process $e^-\gamma \to e^-\tau\mu$ in future lepton collider experiments.

VI. Conclusions

The individual lepton numbers L_e, L_μ, and L_τ are automatically conserved and the tree-level LFV processes are absent in the SM, due to unitary of the leptonic analog of CKM mixing matrix and the masslessness of the three neutrinos. However, the neutrino oscillation data provide very strong evidence for mixing and oscillation of the flavor neutrinos, which imply that the separated lepton numbers are not conserved. Thus, any observation of the effects for the LFV processes would be a clear signature of new physics. This fact and the improvement of the relevant experimental measurements have brought considerable attention to study these processes in the context of specific popular models beyond the SM and see whether the LFV effects can be tested in future high-energy experiments.

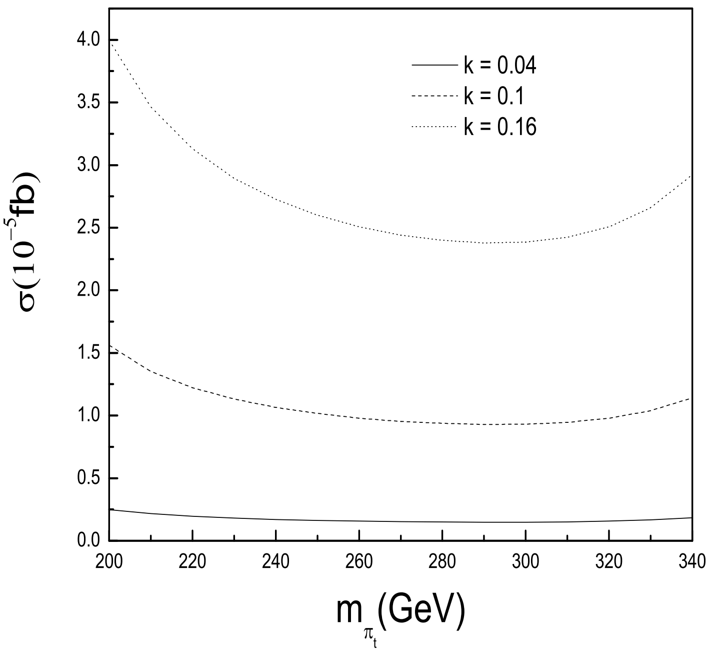

Figure 6: The effective cross section $\sigma(s)$ as a function of m_{π_t} for $\sqrt{s} = 500GeV$, $\varepsilon = 0.05$ and three values of the mixing parameter k.

The topcolor scenario is one of the important candidates for the mechanism of $EWSB$. The presence of physical top-pions in the low-energy spectrum is a common feature of topcolor models. Since topcolor interactions are assumed to couple preferentially to the third generation and thus do not possess GIM mechanism, the physical top-pions have large Yukawa couplings to the third family fermions and can induce the FC scalar couplings, which might give significant contributions to the FC processes. The effects of the top-pion on these processes are governed by its mass m_{π_t} and the relevant flavor mixing factors, which might produce observable signals at future high-energy experiments.

In this paper, we have calculated the contributions of the neutral top-pion π_t^0 predicted by $TC2$ models to the LFV processes $\mu^+\mu^- \to \tau\mu(\tau e), \gamma\gamma \to \tau\mu(\tau e), e^+e^- \to \tau\mu(\tau e)$, and $e\gamma \to e\tau\mu(e\tau e)$, and discussed its possible LFV signals in the future lepton colliders. We find that the value of the cross sections for these LFV processes can indeed be enhanced by several orders of magnitude. With reasonable values of the free parameter, some of these processes may be within the observable threshold of near-future lepton collider

experiments. For example, taking into account the constrains of the present experimental limit of the LFV process $\mu \to e\gamma$ on the mixing factor k and assuming $150GeV \leq m_{\pi_t} < 180GeV$, we find that the cross sections of the LFV processes $\mu^+\mu^- \to \tau\mu$ and $e^+e^- \to \gamma\gamma \to \tau\mu$ can reach $1.5 \times 10^2 fb$ and $8.3 \times 10^{-2} fb$, respectively. However, for $m_{\pi_t} \geq 200GeV$, the cross sections of all of the LFV processes are very small, which cannot be detected in future experiments. Thus, we expect that the light top-pions predicted by topcolor scenario might produce observable LFV signals in future lepton collider experiments.

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